Friction

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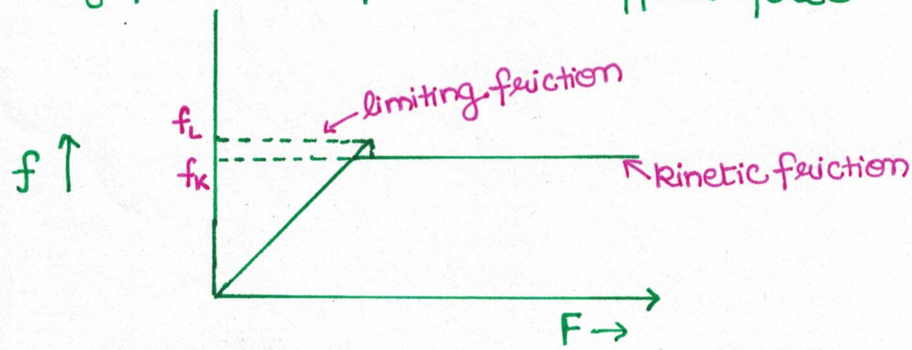
Friction

Variation of frictional force with applied force



LAWS OF DRY FRICTION

1.) The value of limiting friction is directly proportional to the normal reaction b/w the contacting surfaces.
Mathematically,

$$f_L \propto N$$

$$f_L = \mu_s N$$ [μ_s is the coefficient of static friction

The value of μ_s depends on the roughness of contacting surfaces, rougher the surface, higher the μ_s.

2.) The limiting friction is independent of the contacting area.

NOTE: The value of kinetic friction is slightly less than limiting friction and is also proportional to the normal reation.
Mathematically,

$$f_K = \mu_K N$$

μ_K is called coefficient of kinetic friction.

$$\boxed{\mu_K \lesssim \mu_s}$$

ANGLE OF REPOSE

The minimum angle of inclination of a wedge required for slipping of a block is called the angle of repose.

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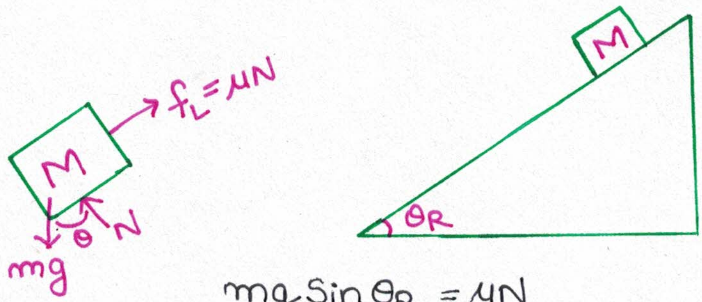
RELATION BETWEEN μ & ANGLE OF REPOSE

FBD



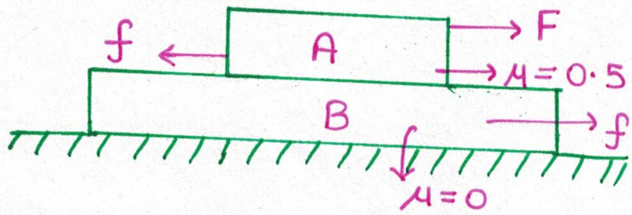
$$mg\sin\theta_R = \mu N$$
$$mg\cos\theta_R = N$$
$$\boxed{\mu_s = \tan\theta_R}$$

NOTE: The friction force before slipping is called Static friction and the frictional force at the verge of slipping is called limiting friction.

FINDING THE DIRECTION & MAGNITUDE OF FRICTIONAL FORCE

The frictional force acts opposite to the tendency of relative motion.

Think what ~~about~~ would have happened if the surface were smooth. Now the relative motion will be clearly visible.



Finding the magnitude:

Whenever in doubt

Step 1: Assume the friction to be static.

Step 2: Find the required value of frictional force for it to be static using Newton's laws. Call it f_R.

Step 3: Compare the required value f_R with limiting friction f_L.

Step 4: If $f_R \leq f_L$ then our assumption is correct and actual frictional force is $f = f_R$.

Step 5: However if $f_R > f_L$ then our assumption is wrong and actual frictional force is $f = f_L$ and $f = f_K$.

(Think of f_L as available friction and f_R as required friction.)

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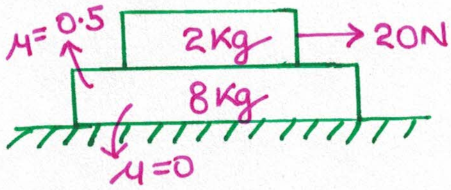
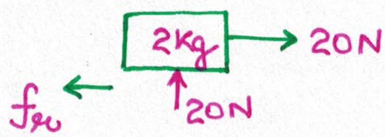
Q.) For the shown system, find the frictional force and the acceleration of the blocks.



assuming static,

$$a = \frac{20}{10} = 2\, m/s^2$$



$$20 - f_r = 2 \times 2$$

$$f_r = 16$$

$$f_L = \mu N = 0.5 \times 20 = 10N$$

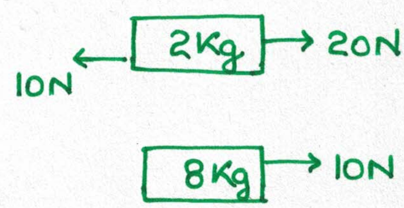
$$f_R > f_L$$

∴ assumption is wrong

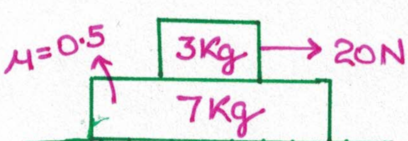
$$f = f_K = 10N$$

$$\vec{a}_A = \frac{10}{2} = 5\, m/s^2$$

$$\vec{a}_B = \frac{10}{8} = 1.25\, m/s^2$$



Q.) Repeat the previous problem for the shown case -



$$a = 2\, m/s^2$$

$$f_L = 0.5 \times 30 = 15N$$

$$2 \times 3 = 20 - f_r$$

$$f_r = 14N$$

$$\therefore f = 14N$$

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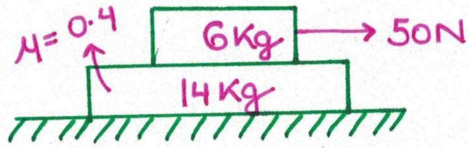
Q.) Repeat the previous problem for the shown case



$a = \frac{5}{2} m/s^2$

$f_L = \mu N$
$= 24N$

$15 = 50 - f_R$

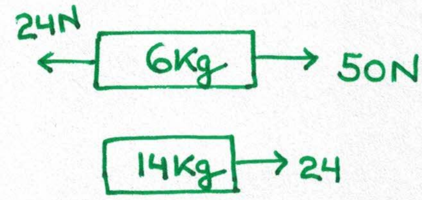
$f_R = 35N$

$f_L < f_R$

$f = 24N$

$\vec{a}_A = \frac{26}{6} m/s^2 = \frac{13}{3} m/s^2$

$\vec{a}_B = \frac{24}{14} m/s^2 = \frac{12}{7} m/s^2$



2nd Method (Assuming Kinetic friction)

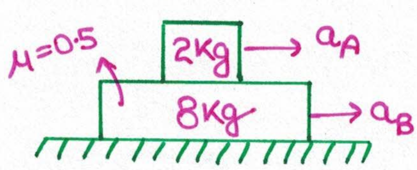
Step 1: Assume friction to be kinetic.

Step 2: Find the magnitude and direction of acceleration of the block using Newton's laws.

Step 3: At every interface, ensure that the direction of relative motion is opposite to the friction. If it is not then our assumption is correct.

Step 4: If assumption is wrong, then solve the problem using static friction.

Q.) Repeat the previous problem.

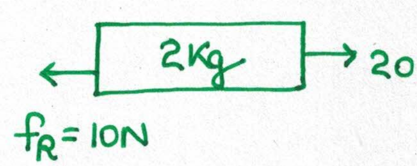


$a_{AB} = \longrightarrow$

$a_{BA} = \longleftarrow$

$a_A = 5 m/s^2$

$a_B = 1.25 m/s^2$



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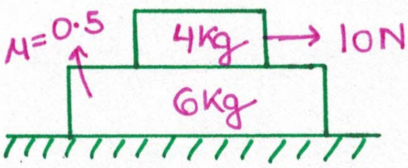
Q.) Repeat the previous problem

$$a_3 = \frac{5}{3}\,m/s^2 = 1.67\,m/s^2$$

$$a_7 = \frac{15}{7}\,m/s^2 = 2.14\,m/s^2$$

$$a_{37} = \longleftarrow \qquad , a_{73} = \longrightarrow$$

Q.) Repeat the previous problem



$$a_4 = -\frac{5}{2}\,m/s^2 = -2.5\,m/s^2 \;(\longleftarrow)$$

$$a_6 = \frac{10}{3}\,m/s^2$$

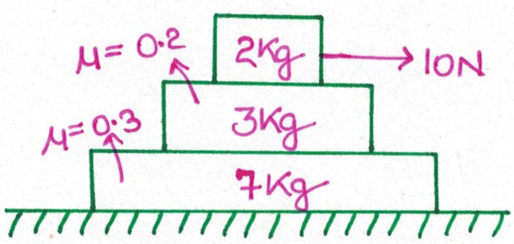
$$a_{46} = \longleftarrow \qquad , a_{64} = \longrightarrow$$ (Condradiction)

$$a = 1\,m/s^2$$

$$4 = 10 - f_R$$

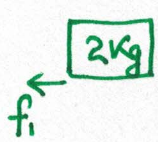
$$f_R = 6N$$

Q.) Repeat the previous problem



$$a = \frac{5}{6}\,m/s^2$$

$$10 - f_1 = 2 \times \frac{5}{6}$$



$$f_1 = 10 - \frac{5}{3} = \frac{25}{3} = 8.33N$$

$$f_2 = 2\cancel{0} \times \frac{2}{1\cancel{0}} = 4N$$

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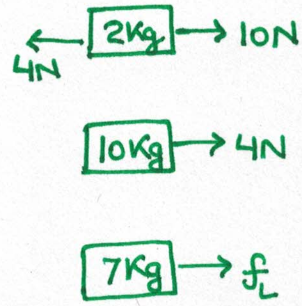
∴ assumption is wrong

let it be kinetic & static



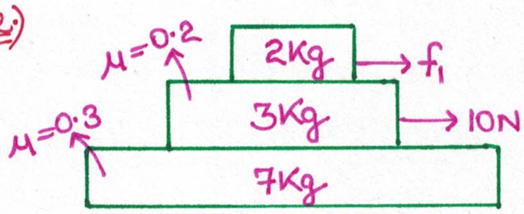
$$a_A = 3\,m/s^2$$

$$a_{BC} = 0.4\,m/s^2$$

$$f_2 = 7 \times 0.4 = 2.8N$$

$$f_{L2} = 0.3 \times 50 = 15N$$

Que.)



Static

Static

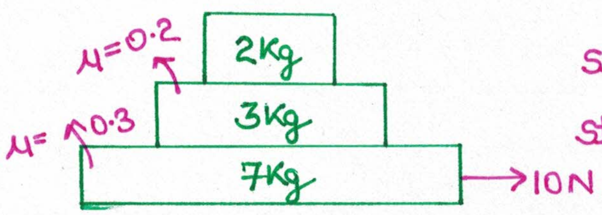
$$f_1 = 2 \times \frac{5}{6} = \frac{5}{3}N$$

$$f_{L_1} = 2\cancel{0} \times \frac{2}{1\cancel{0}} = 4N$$

$$f_2 = 7 \times \frac{5}{6} = \frac{35}{6}N$$

$$f_{L2} = 15N$$

Que.)



Static

Static

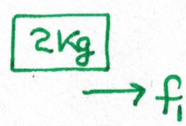
$$a = \frac{5}{6}\,m/s^2$$

$$f_1 = \frac{25}{6}N$$

$$f_{L1} = 15N$$

$$f_2 = 2 \times \frac{5}{6} = \frac{5}{3}N$$

$$f_{L2} = 4N$$



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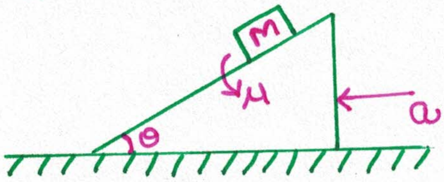
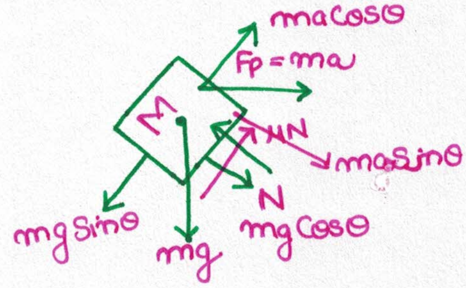
Que.) For the shown system find the range of acceleration for which the block does not slip relative to the wedge.





$$Mg\sin\theta = Ma\cos\theta + \mu N$$

$$(N = mg\cos\theta + Ma\sin\theta) \times \mu$$

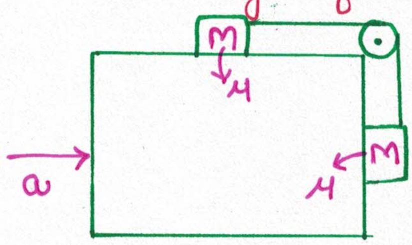
$$Mg\sin\theta - \mu Mg\cos\theta = Ma\cos\theta + \mu Ma\sin\theta$$

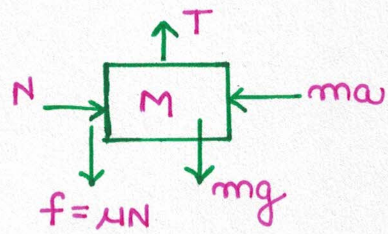
$$a = g\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)$$

$$a_{min} = g\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)$$

$$a_{max} = g\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right) \quad (\mu N \text{ will negative})$$

Que.) Find the range for which system remains in equilibrium.





For a_{max}

$$T = mg + \mu N$$

$$N = ma$$

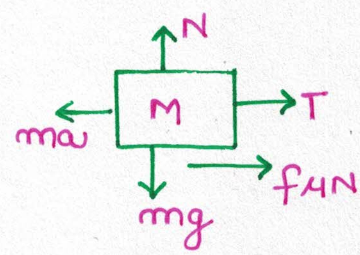
$$T = mg + \mu Ma$$

$$a_{max} = \frac{T - mg}{M\mu} \quad , \quad a_{min.} = \frac{mg - T}{m\mu}$$

$$T + \mu N = ma$$

$$T = ma - \mu N$$

$$a_{max} = \frac{ma - \mu N - mg}{M\mu}$$



$$a_{max} = \frac{mg - \mu mg - ma}{m\mu}$$

$$\cancel{m}\mu a = \cancel{m}g - \mu\cancel{m}g - \cancel{m}a$$

$$a_{max} = \frac{g - \mu g}{1 + \mu}$$

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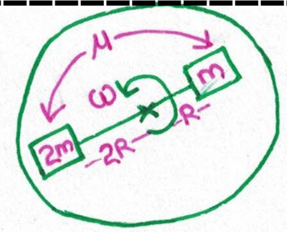
Que.)



$$\therefore 3M\omega^2R = 3\mu Mg$$

$$\omega = \sqrt{\frac{\mu g}{R}}$$

PULLING FORCE METHOD

Pulling force method can be used to find the acceleration of the chain when every element of the chain is moving with the same magnitude of acceleration.

Step 1 Choose the sense of the chain.

Step 2 Find the components of all the external forces along the chain (If the component is against the sense of chain then take it as negative.)

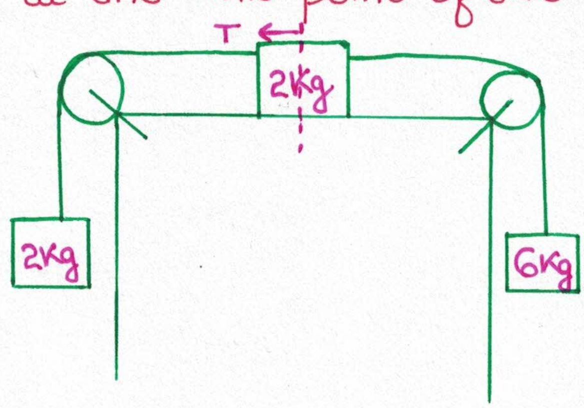
Step 3 Now add these components to get net pulling force.

Step 4 Now acceleration of chain is not pulling force divided by net mass.

Que.) For the shown system, find

(i) acceleration of chain

(ii) Tension at the mid point of the top brick. (given $\mu = 0$)



(i) $a = \frac{40}{10} = 4\,m/s^2$

(ii) $60 - T = 7 \times 4$

$T = 60 - 28$

$T = 32N$

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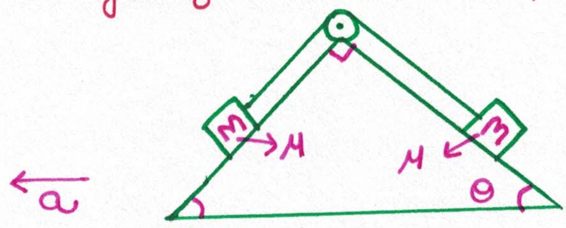
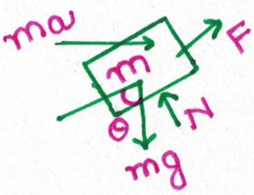
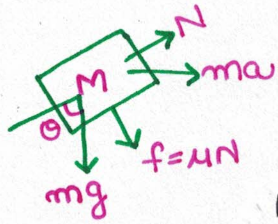
Que) For what range of a will this system not slip?





$$N_1 = ma\cos\theta + mg\sin\theta$$



$$N_2 + ma\sin\theta = mg\cos\theta$$

$$Mg\cos\theta - \mu(mg\sin\theta + ma\cos\theta) - Ma\sin\theta$$
$$- Ma\cos\theta - Mg\sin\theta - \mu(Mg\cos\theta - Ma\sin\theta) = 0$$

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